Nonlinear Analysis 74 (2011) 3876–3883

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Long-time behavior of reaction–diffusion equations with dynamical boundary condition[☆]

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ARTICLE INFO

Article history:
Received 15 April 2010
Accepted 14 February 2011

MSC:
37L05
35B40
35B41

Keywords:
Reaction–diffusion equation
Dynamical boundary condition
Asymptotic regularity
Attractors

ABSTRACT

In this paper, we study the long-time behavior of the reaction–diffusion equation with dynamical boundary condition, where the nonlinear terms f and g satisfy the polynomial growth condition of arbitrary order. Some asymptotic regularity of the solution has been proved. As an application of the asymptotic regularity results, we can not only obtain the existence of a global attractor $\mathscr{A}$ in $(H^1(\Omega) \cap L^p(\Omega)) \times L^q(\Gamma)$ immediately, but also can show further that $\mathscr{A}$ attracts every $L^2(\Omega) \times L^2(\Gamma)$-bounded subset with $(H^1(\Omega) \cap L^{p+\delta}(\Omega)) \times L^{q+\kappa}(\Gamma)$-norm for any $\delta, \kappa \in [0, \infty)$.

1. Introduction

In this paper, we consider the asymptotic behavior of the solution of the following reaction–diffusion equation with dynamical boundary condition:

$$\begin{cases} u_t - \Delta u + f(u) = h(x), & \text{in } \Omega, \\ u_t + \dfrac{\partial u}{\partial \nu} + g(u) = 0, & \text{on } \Gamma, \\ u(x,0) = u_0(x), \end{cases} \tag{1.1}$$

where Ω is a bounded domain of $\mathbb{R}^N (N \geqslant 3)$ with a smooth boundary Γ, $h(x) \in L^2(\Omega)$. The functions f and $g \in C^1(\mathbb{R}, \mathbb{R})$, satisfy the following conditions:

$$C_1|s|^p - k_1 \le f(s)s \le C_2|s|^p + k_2, \quad p > 2, \tag{1.2}$$

$$C_3|s|^q - k_3 \le g(s)s \le C_4|s|^q + k_4, \quad q > 2, \tag{1.3}$$

and

$$f'(s) \ge -l, \quad g'(s) \ge -m, \tag{1.4}$$

here $l, m > 0$, $C_i > 0$, $k_i > 0$, $i = 1, 2, 3, 4$.

☆ This work was supported by the NSFC Grant 10901063, the Mathematical Tianyuan Foundation of China Grant 10926089 and the Fund of Physics & Mathematics of Lanzhou University Grant LZULL200903.

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doi:10.1016/j.na.2011.02.022