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Solitary waves for the Klein–Gordon–Maxwell system with critical exponent[☆]

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ABSTRACT

In this paper we study the existence of solutions for nonlinear Klein–Gordon–Maxwell equations coupled with Maxwell's equations when the nonlinearity exhibits critical growth. We improve some previous existence results in Azzollini et al. (2009) [5], Carrião et al. (2009) [4] and Cassani (2004) [3].

1. Introduction

Recently, as a follow-up to [1], a certain number of works have been devoted to the Klein–Gordon–Maxwell system

$$\begin{cases} -\Delta u + [m^2 - (e\phi - \omega)^2]u = f(u), & \text{in } \mathbb{R}^3, \\ \Delta\phi = e(e\phi - \omega)u^2, & \text{in } \mathbb{R}^3, \end{cases} \tag{Pf}$$

where m, e and ω are real constants. Problem (Pf) concerns certain kinds of solitary charged waves in nonlinear equations of Klein–Gordon or Schrödinger type.

In the following, we review some assumptions on problem (Pf), and the corresponding results. In [2], under the conditions

(DM_1) $f : \mathbb{R} \to \mathbb{R}$ is a continuous function, and
(DM_2) for every $s \in \mathbb{R}$, either $f(s)s + 2(m^2 - \omega^2)s^2 \geq 6F(s)$ or $2F(s) \geq f(s)s$,

D'Aprile and Mugnai proved that any weak solution $(u, \phi) \in H^1(\mathbb{R}^3) \times D^{1,2}(\mathbb{R}^3)$ of (Pf) vanishes identically. In particular, $f(u) = |u|^{2^*-2}u$ and $m > \omega$ satisfy (DM_1) and (DM_2). Notice that the critical Sobolev exponent $2^* = 6$ in dimension three. In [3], Cassani considered problem (Pf) with $f(u) = \mu|u|^{q-2}u + |u|^{2^*-2}u$ (adding a lower-order perturbation), where $\mu > 0$ and $q \in [4, 6)$. He showed the existence of nontrivial solutions of (Pf), provided one of the following conditions is satisfied:

(i) $q \in (4, 6)$, $|m| > |\omega| > 0$ and $\mu > 0$,
(ii) $q = 4$, $|m| > |\omega| > 0$ and $\mu > 0$ sufficiently large.

Later, Carrião et al. [4] improved the result of Cassani [3]. They assume that one of the following conditions is satisfied:

(i) $q \in (4, 6)$, $|m| > |\omega| > 0$ and $\mu > 0$,
(ii) $q = 4$, $|m| > |\omega| > 0$ and $\mu > 0$ sufficiently large.
(iii) $q \in (2, 4)$, $|m|\sqrt{q-2} > |\omega|\sqrt{2} > 0$ and $\mu > 0$ sufficiently large.

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