Nonlinear Analysis 74 (2011) 2862–2868

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Existence of real eigenvalues of real tensors

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ARTICLE INFO

Article history:
Received 29 November 2010
Accepted 13 January 2011

MSC:
primary 15A18
15A69

Keywords:
Multilinear algebra
Higher order tensor
Eigenvalues

ABSTRACT

We use the Brouwer degree to establish the existence of real eigenpairs of higher order real tensors in various settings. Also, we provide some finer criteria for the existence of real eigenpairs of two-dimensional real tensors and give a complete classification of the Brouwer degree zero and ± 2 maps induced by general third order two-dimensional real tensors.

1. Introduction

Let $\mathbb{R}$ be the real field, we consider an m-order n-dimensional tensor $\mathcal{A}$ consisting of n^m entries in $\mathbb{R}$:

$$\mathcal{A} = (a_{i_1\cdots i_m}), \qquad a_{i_1\cdots i_m} \in \mathbb{R}, \quad 1 \le i_1, \ldots, i_m \le n.$$

To an n-vector $x = (x_1, \ldots, x_n)$, real or complex, we define an n-vector:

$$\mathcal{A}x^{m-1} := \left(\sum_{i_2,\ldots,i_m=1}^{n} a_{ii_2\cdots i_m} x_{i_2} \cdots x_{i_m} \right)_{1\le i\le n}.$$

The following were first introduced and studied by Qi [1–3] and Lim [4].

Definition 1.1. Let $\mathcal{A}$ be an m-order n-dimensional real tensor. Assume that $\mathcal{A}x^{m-1}$ is not identically zero. We say $(\lambda, x) \in \mathbb{C} \times (\mathbb{C}^n \setminus \{0\})$ is an eigenpair if they satisfy the equation $\mathcal{A}x^{m-1} = \lambda x^{[m-1]}$, where $x^{[m-1]} = (x_1^{m-1}, \ldots, x_n^{m-1})$. We say it is an H-eigenpair if they are both real.

Definition 1.2. Let $\mathcal{A}$ be an m-order n-dimensional real tensor. Assume that $\mathcal{A}x^{m-1}$ is not identically zero. We say $(\lambda, x) \in \mathbb{C} \times (\mathbb{C}^n \setminus \{0\})$ is an E-eigenpair if they satisfy the equation $\mathcal{A}x^{m-1} = \lambda x$. We say it is a Z-eigenpair if they are both real.

The above notions of eigenvalues were generalized by [5] as follows.

Definition 1.3. Let $\mathcal{A}$ and $\mathcal{B}$ be two m-order n-dimensional real tensors. Assume that both $\mathcal{A}x^{m-1}$ and $\mathcal{B}x^{m-1}$ are not identically zero. We say $(\lambda, x) \in \mathbb{C} \times (\mathbb{C}^n \setminus \{0\})$ is an eigenvalue–eigenvector of $\mathcal{A}$ relative to $\mathcal{B}$, if the n-system of

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doi:10.1016/j.na.2011.01.008