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The number of periodic solutions of some analytic equations of Abel type[☆]

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ABSTRACT

New results are proved to estimate the number of periodic solutions of a differential equation of Abel type by using a modification of a technique introduced by Ilyashenko. The main tool is an estimate on the number of zeros of a holomorphic function. A concrete example is analyzed but the results are presented to make the method flexible and applicable to other equations.

1. Introduction

In [1], Ilyashenko considered the differential equation of polynomial type

$$x' = x^n + \sum_{j=0}^{n-1} a_j(t)x^j \tag{1}$$

with $a_j : \mathbb{R} \longrightarrow \mathbb{R}$ continuous and 1-periodic. Assuming that all the coefficients $a_j(t)$ were dominated by a common bound $C \geq 1$, he obtained an estimate (depending on n and C) on the number of periodic solutions of Eq. (1). The purpose of this paper is to show that this method can also be applied to more general equations. As a model, we will consider the equation

$$x' = x^3 + \sin x + p(t), \qquad p(t+1) = p(t) \tag{2}$$

and we will obtain an estimate on the number of periodic solutions depending only on

$$\|p\|_\infty = \max_{t\in[0,1]} |p(t)| \leq C. \tag{3}$$

The basic tool employed in [1] is the so-called Jensen's Lemma. This is a result in Complex Analysis that allows to estimate the number of zeros of a holomorphic function in a domain D in terms of the behavior on the boundary ∂D. The standard version of this lemma assumes that D is a disk, but the result can be transported to other domains via Riemann's Theorem on conformal mappings. The approach in [1] was to consider certain domains with the shape of a stadium and to employ ideas taken from hyperbolic geometry to estimate some quantities related to the Riemann's mapping for these domains. Our approach will be more straightforward; we will consider the explicit Christoffel–Schwarz formula mapping the unit disk onto a rectangle. This will allow us to state a version of Jensen's Lemma for the rectangle, where all the quantities

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