Some generalizations of locally and weakly locally uniformly convex space

Z.H. Zhang *, C.Y. Liu

College of Fundamental Studies, Shanghai University of Engineering Science, Shanghai, 201620, PR China

A R T I C L E   I N F O

Article history:
Received 22 December 2009
Accepted 16 February 2011

Keywords:
Strongly convex space
Very convex space
Almost locally uniformly rotund space
Weakly almost locally uniformly rotund space
k-strongly convex space
k-very convex space
Strongly Chebyshev set

A B S T R A C T

In this paper, we prove that strongly convex space and almost locally uniformly rotund space, very convex space and weakly almost locally uniformly rotund space are respectively equivalent. We also investigate a few properties of k-strongly convex space and k-very convex space, and discuss the applications of strongly convex space and very convex space in approximation theory.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

In 1977, Sullivan [1] defined very convex space (also called very rotund space). Wu and Li [2] introduced and studied the strongly convex space. Many results of these two classes of convexities were discussed in [3–6]. In 2000, He [7] gave the definition of the k-strongly convex space, which is a generalization of the strongly convex space. 1-strongly convex space is equivalent to strongly convex space, and k-strongly convex space implies (k + 1)-strongly convex space. However, the converse is not generally true. Many results of k-strongly convex space were obtained in [7,8]. Recently, the author [9] discussed the important applications of the strongly convex space and the very convex space in approximation theory, and obtained some good results. In 2000, Bandgapadhyay et al. [10] proposed two generalizations of locally uniformly rotund space, which are called almost locally uniformly rotund space and weakly almost locally uniformly rotund space. Many results of these two classes of convexities were studied in [11–13]. In this paper, we will prove that almost locally uniformly rotund space is equivalent to strongly convex space and that weakly almost locally uniformly rotund space is equivalent to very convex space. Thus, we unify the results of the studies about the strongly convex space (resp. very convex space) and the almost locally uniform rotundity (resp. weakly almost locally uniform rotundity). We introduce k-very convex space, which is a generalization of very convex space, and show that 1-very convex space is equivalent to very convex space, k-very convex space implies (k + 1)-very convex space. But, the converse is not generally true. We also investigate some properties of k-strongly convex and k-very convex spaces, and show that Banach space X is strongly convex (resp. very convex) if and only if every proximinal convex set of X is norm-strongly Chebyshev (resp. weak-strongly Chebyshev).

* Corresponding author. Tel.: +86 21 67791195.
E-mail address: zhz@sues.edu.cn (Z.H. Zhang).

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.