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On the stability of gradient polynomial systems at infinity

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ABSTRACT

We first define the notion of the infimum at infinity of a polynomial function and the notion of stability at infinity near the fiber of the gradient descent system. Then we prove that the gradient descent system is stable at infinity near the fiber of the infimum value at infinity.

1. Introduction

Let f be a real, continuously differentiable function on $\mathbb{R}^n$ and consider the gradient system

$$\dot{x}(t) = -\nabla f(x(t)), \tag{1.1}$$

where $\nabla f(x)$ denotes the Euclidean gradient of f at x. A classical result [1, Theorem 3.1] states that: if z is an isolated minimum point of f, then z is an asymptotically stable equilibrium point of gradient system (1.1).

Recently, by using the Łojasiewicz gradient inequality for analytic functions, Absil and Kurdyka [2] proved that z is a stable (not necessarily asymptotically stable) equilibrium point of gradient system (1.1) if and only if it is a local (not necessarily isolated) minimum of f.

In this note, we investigate the stability at infinity of (1.1) when f is a polynomial. We first define the notion of the infimum value at infinity of a polynomial function and the notion of stability at infinity near the fiber of the gradient descent system. Then we prove that:

(i) If y_0 is an isolated infimum value at infinity of f then (1.1) is asymptotically stable at infinity near the fiber $f^{-1}(y_0)$;
(ii) If y_0 is an infimum value at infinity (not necessarily isolated) of f then (1.1) is stable at infinity (not necessarily asymptotically stable) near the fiber $f^{-1}(y_0)$.

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