Applying a fixed point theorem of Krasnosel'skii type to the existence of asymptotically stable solutions for a Volterra–Hammerstein integral equation

Le Thi Phuong Ngoc a, Nguyen Thanh Long b,∗

a Nhatrang Educational College, 01 Nguyen Chanh Str., Nhatrang City, Viet Nam
b Department of Mathematics and Computer Science, University of Natural Science, Vietnam National University HoChiMinh City, 227 Nguyen Van Cu Str., Dist.5, HoChiMinh City, Viet Nam

A R T I C L E   I N F O

Article history:
Received 3 February 2011
Accepted 9 March 2011
Accepting Editor: Ravi Agarwal

MSC:
47H10
45G10
47N20
65J15

Keywords:
The fixed point theorem of Krasnosel’skii type
Volterra–Hammerstein integral equation
Contraction mapping
Completely continuous
Asymptotically stable solution

A B S T R A C T

Using a fixed point theorem of Krasnosel’skii type, the paper proves the existence of asymptotically stable solutions for a Volterra–Hammerstein integral equation.

© 2011 Elsevier Ltd. All rights reserved.

1. Introduction

In this paper, we consider the following nonlinear integral equation

\[ x(t) = q(t) + f(t, x(t)) + \int_0^t V(t, s, x(s))ds + \int_0^\infty G(t, s, x(s))ds, \quad t \in \mathbb{R}_+, \]  

(1.1)

where \( E \) is a Banach space with norm \( \| \cdot \| \), \( \mathbb{R}_+ = [0, \infty) \), \( q : \mathbb{R}_+ \to E ; f : \mathbb{R}_+ \times E \to E ; G : \mathbb{R}_+ \times \mathbb{R}_+ \times E \to E ; V : \Delta \times E \to E \) are supposed to be continuous and \( \Delta = \{(t, s) \in \mathbb{R}_+ \times \mathbb{R}_+ : s \leq t\} \).

We call the integral equation (1.1) to be the Volterra–Hammerstein integral equation because the right-hand side consists of an integral of Volterra type and an integral of Hammerstein type, (see [1], p. 151). The integral equation (1.1) also is called to be an integral equation of mixed type; see [2].