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Stability of perturbed n-dimensional Volterra differential equations

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ABSTRACT

In this work we deal with the problem of the stability and uniform stability of the perturbed n-dimensional Volterra integral and differential equation

$$x'(t) = A(t)x(t) + \int_{-\infty}^{t} C(t,s)x(s)\,\mathrm{d}s + \int_{-\infty}^{t} D(t,s)x'(s)\,\mathrm{d}s + b(t).$$

Some sufficient conditions for the zero solution of this equation to be stable as well as uniformly stable have been obtained.

1. Introduction

In recent years, there has been increasing interest in the Volterra system; see [1–5]. Many scholars and researchers have considered the stability of the Volterra integral–differential equation

$$\dot{x}(t) = A(t)x(t) + \int_{0}^{t} C(t,s)x(s)\mathrm{d}s. \tag{1}$$

In the above equation, $A(t)$ is an $n\times n$ function matrix and is continuous on the interval $[0,\infty)$, $C(t,s)$ is an $n\times n$ matrix, and when $0 \le s \le t < \infty$ it is continuous, and $\int_t^{+\infty} \|C(u,s)\|\mathrm{d}u$ is continuous in t when $0 \le s \le t < \infty$. In [1] the sufficient conditions for the stability and uniform stability of the zero solution when $A(t)$ is a scalar function and a constant matrix respectively were discussed. [2] improved the corresponding theorem from [1] and considered the condition when matrix A was a function matrix, and [3] gave a series of sufficient conditions for the stability, uniform stability and asymptotic stability of the zero solution of Eq. (1) when $A(t)$ is an n-dimensional function matrix. Theorem 1 in [3] is the following: If there exists a matrix P, with $K_1 \ge \|P\|$ and $K_2 \ge 0$, such that

$$2\lambda_M(t,P,A) + K_1\int_0^t \|C(t,s)\|\mathrm{d}s + \int_t^{+\infty} \|C(u,t)\|\mathrm{d}u \le -K_2,$$

then the zero solution of Eq. (1) is stable, where $\lambda_M(t,p,A)$ is the biggest eigenvalue of the matrix

$$\frac{PA(t) + A^T(t)P}{2}.$$

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