Available online at www.sciencedirect.com
SciVerse ScienceDirect
Mathematics and Computers in Simulation 82 (2012) 2253–2267
MATHEMATICS AND COMPUTERS IN SIMULATION
www.elsevier.com/locate/matcom

Original article

Mathematical modeling of the flow in a pipeline with a leak

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Received 25 May 2009; received in revised form 7 June 2012; accepted 14 June 2012
Available online 23 June 2012

Abstract

Pipeline leaks can lead to excessive economical loss as well as posing environmental hazards. Mathematical modeling of the flow in a pipeline with a leak can be used to evaluate the loss caused by a leak and provide a guide for pipeline operation as well as an aid for leak detection. The mathematical expressions describing the flow in a pipeline with a leak have been formulated. A new approach based on the analogy between pipeline and electric circuit has been proposed to solve these equations. In this approach, an equivalent circuit with a certain structure has been used to simulate the leaking pipeline and the flow characteristics of the pipeline end have been treated with approximation. The mathematical model of the pipeline with a leak has been developed with the solution of the equations. The model was verified by using an experimental gas pipeline and a real oil pipeline. The simulated results agreed well with the experimental recordings confirmed the accuracy of the models and the validity and efficiency of the proposed approach.

Keywords: Pipeline; Mathematical modeling; Hydraulic–electric analogy; Leakage; Equivalent circuit

1. Introduction

The operation of pipelines carrying petroleum products and chemicals is of prime concern from the safety and economic points of view. Pipeline leaks, which result from bad workmanship or a destructive cause, can lead to excessive economical loss as well as posing environmental hazards. Mathematical modeling of the flow in a gas or oil pipeline with a leak can accurately predict the variations of flow variables such as pressures and flow rates at the ends of a pipeline following the occurrence of a leak. Thus it can be used to evaluate the loss caused by a leak and provide a guide for pipeline operation. Moreover, the model can generate test data for validating the model based leak detection and location methods [1,2,4]. Most of these models were verified by pipeline simulators, but the accuracy of the pipeline simulators has not been validated by the real cases.

The equations governing the behavior of the flow in a pipeline constitute a set of partial differential equations. They are difficult to be solved analytically and numerical methods such as the method of characteristics (MOC) and a variety of explicit and implicit finite difference schemes [8,18] have to be used. All these numerical methods require information of the boundary conditions of the pipeline, which depend on the characteristics of upstream and downstream equipment. Hence the model of a leaking pipeline becomes less general and increases in complexity when considering these equipment. Oke et al. [16] simulated the transient induced by a pipeline rupture followed by a shut down of an emergency valve, i.e., downstream conditions were changed in a prescribed manner. Nie et al. [15] simulated the

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http://dx.doi.org/10.1016/j.matcom.2012.06.007