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Computational investigations of scrambled Faure sequences

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Abstract

The Faure sequence is one of the well-known quasi-random sequences used in quasi-Monte Carlo applications. In its original and most basic form, the Faure sequence suffers from correlations between different dimensions. These correlations result in poorly distributed two-dimensional projections. A standard solution to this problem is to use a randomly scrambled version of the Faure sequence. We analyze various scrambling methods and propose a new nonlinear scrambling method, which has similarities with inversive congruential methods for pseudo-random number generation. We demonstrate the usefulness of our scrambling by means of two-dimensional projections and integration problems.

Keywords: Faure sequence; Low-discrepancy sequences; (quasi)-Monte Carlo; Linear scrambling; Nonlinear scrambling

1. Introduction

The term ‘Monte Carlo (MC) method’ is often used to refer to a well-known family of stochastic algorithms and techniques for solving a wide variety of problems. It is well-known that the probabilistic error for these Monte Carlo methods converges as $\mathcal{O}(N^{-1/2})$ if information about regularity (or smoothness) is not used. Here, N is the number of sample points used. So-called ‘quasi-Monte Carlo (qMC) methods’ [21], based on deterministic point-sets or sequences, form an alternative to MC methods and lead to smaller approximation errors in many practical situations. While quasi-random numbers do improve the convergence of applications like numerical integration, it is by no means trivial to provide practical error estimates in qMC due to the fact that the only rigorous error bounds, provided via the Koksma–Hlawka inequality, are very hard to utilize. In fact, the common practice in MC of using a predetermined error criterion as a deterministic termination condition, is almost impossible to achieve in qMC without extra technology. In order to provide such dynamic error estimates for qMC methods, several researchers [27,23] proposed the use of Randomized qMC (RqMC) methods [14], where randomness can be brought to bear on quasi-random sequences through scrambling and other related randomization techniques [30,3]. One can rigorously show [16] that under relatively loose conditions each of the randomized qMC rules are statistically independent and thus can be used to form a traditional MC error estimate using confidence intervals based on the sample variance. The core of randomized qMC is a fast and effective algorithm to randomize (scramble) quasi-random sequences.

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