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Computing quasi-interpolants from the B-form of B-splines ☆

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Abstract

In general, for a sufficiently regular function, an expression for the quasi-interpolation error associated with discrete, differential and integral quasi-interpolants can be derived involving a term measuring how well the non-reproduced monomials are approximated. That term depends on some expressions of the coefficients defining the quasi-interpolant, and its minimization has been proposed. However, the resulting problem is rather complex and often requires some computational effort. Thus, for quasi-interpolants defined from a piecewise polynomial function, φ, we propose a simpler minimization problem, based on the Bernstein–Bézier representation of some related piecewise polynomial functions, leading to a new class of quasi-interpolants.

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1. Introduction and notations

Let φ be a s-variate B-spline on a uniform partition τ, i.e. a compactly supported nonnegative piecewise polynomial function defined on $\mathbb{R}^s$, $s \geq 1$, and suppose φ is normalized by the condition $\sum_{i\in\mathbb{Z}^s}\varphi(\cdot - i) = 1$. Quasi-interpolants

$$Qf := Q(f) := \sum_{i\in\mathbb{Z}^s} \lambda f(\cdot + i)\,\varphi(\cdot - i) \qquad (1)$$

are useful tools in practice since they provide approximating splines from the available information of the function to be approximate without solving any linear system. The monographs [4,6,13] provide detailed description of standard methods for constructing such approximants. In general, if we know the values at the integer points of the function and its partial derivatives up to the appropriate order, we can consider the differential operator associated with the Taylor expansion of $1/\hat{\varphi}$, $\hat{\varphi}$ denoting the Fourier transform of φ, and this operator provides the optimal approximation order of the space $\mathcal{S}(\varphi) := \mathrm{span}(\varphi(\cdot - i))_{i\in\mathbb{Z}^s}$ spanned by the shifts of φ, as well as good results for the error $Ef := f - Qf$. A differential quasi-interpolant is obtained and the linear functional λ acting on a given function g is a linear combination

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