

## Original article

Inverse problem of time-dependent heat sources numerical reconstruction<sup>☆</sup>Liu Yang<sup>a,\*</sup>, Mehdi Dehghan<sup>b</sup>, Jian-Ning Yu<sup>a</sup>, Guan-Wei Luo<sup>a</sup><sup>a</sup> Department of Mathematics, Lanzhou Jiaotong University, Lanzhou, Gansu 730070, People's Republic of China<sup>b</sup> Department of Applied Mathematics, Faculty of Mathematics and Computer Sciences, Amirkabir University of Technology, No. 424 Hafez Avenue, Tehran, Iran

Received 15 December 2009; received in revised form 3 January 2011; accepted 12 January 2011

Available online 31 January 2011

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**Abstract**

This work studies the inverse problem of reconstructing a time-dependent heat source in the heat conduction equation using the temperature measurement specified at an internal point. Problems of this type have important applications in several fields of applied science. By the Green's function method, the inverse problem is reduced to an operator equation of the first kind which is known to be ill-posed. The uniqueness of the solution for the inverse problem is obtained by the contraction mapping principle. A numerical algorithm on the basis of the Landweber iteration is designed to deal with the operator equation and some typical numerical experiments are also performed in the paper. The numerical results show that the proposed method is stable and the unknown heat source is recovered very well.

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*Mathematics subject classification (2000):* 35R30; 49J20*Keywords:* Inverse problem; Heat source; Green function; Landweber iteration; Numerical results

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**1. Introduction**

In recent years, the inverse problem of determining the coefficient in parabolic equations from some additional conditions has attracted considerable interest. Some detailed treatments of problems in these areas can be found in [2,4,6,7,3,9,10,31,8].

In this paper, we study an inverse problem of recovering the time-dependent source term of heat equation when the extra condition specified at an internal point is given. Problems of this type have important applications in several fields of applied science. The problem can be stated in the following form:

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<sup>☆</sup> Supported by NNSF of China (No. 11061018,10972095), NSF of Gansu Province of China (No. 0916RJZA046) and Fundamental Research Funds for Universities in the Gansu Province of China (No. 620004).

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