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Original article

Center conditions and bifurcation of limit cycles at three-order nilpotent critical point in a septic Lyapunov system ☆

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Abstract

In this paper, center conditions and bifurcation of limit cycles at the nilpotent critical point in a class of septic polynomial differential systems are investigated. With the help of computer algebra system MATHEMATICA, the first 13 quasi-Lyapunov constants are deduced. As a result, sufficient and necessary conditions in order to have a center are obtained. The result that there exist 13 small amplitude limit cycles created from the three order nilpotent critical point is also proved. Henceforth we give a lower bound of cyclicity of three-order nilpotent critical point for septic Lyapunov systems.

MSC: 34C05; 34C07

Keywords: Three-order nilpotent critical point; Center-focus problem; Bifurcation of limit cycles; Quasi-Lyapunov constant

1. Introduction

The nilpotent center problem was investigated by Moussu [16] and Stróżyna [17]. Nevertheless, given an analytic system with a monodromic point, it is still very difficult to detect if it is a focus or a center, even in the case of a concrete polynomial systems. In this paper, we consider an autonomous planar ordinary differential equation having a three-order nilpotent critical point with the form

$$\begin{aligned}\frac{dx}{dt} &= y + y^2 - x^2y + a_{12}xy^2 + a_{50}x^5 + a_{05}y^5 + a_{06}y^6 + 6b_{06}xy^5 \\ &\quad + a_{33}x^3y^3 + \frac{5}{2}b_{15}x^2y^4 + a_{42}x^4y^2 + a_{07}y^7 + a_{52}x^5y^2 + a_{34}x^3y^4, \\ \frac{dy}{dt} &= -2x^3 - a_{12}x^2y + b_{03}y^3 - 5a_{50}x^4y + b_{60}x^6 - b_{06}y^6 - \frac{3}{4}a_{33}x^2y^4 \\ &\quad - b_{15}xy^5 - \frac{4}{3}a_{42}x^3y^3 + b_{34}x^3y^4,\end{aligned} \tag{1.1}$$

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