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Original article

Product quasi-interpolation method for weakly singular integral equation eigenvalue problem ☆

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Abstract

A discrete method of accuracy $O(h^m)$ is constructed to solve the integral equation eigenvalue problem $\lambda\phi(x) = \int_{-1}^{1} K(x, y)\phi(y)dy$, where $K(x, y) = \log|x - y|$ or $K(x, y) = |x - y|^{-\alpha}$, $0 < \alpha < 1$, $\phi(x)$ being the eigenfunction associated with the eigenvalue λ. The method is based first on improving the boundary behavior of the exact solution $\phi(x)$ with the help of a change of variables, and second on the product integration method based on a discrete spline quasi-interpolant (abbr. dQI) of order m.

Keywords: Eigenvalue problem; Spline quasi-interpolation; Product integration methods

1. Introduction

The numerical solution of the integral equation eigenvalue problem

$$\lambda\phi(x) = \int_{-1}^{1} K(x, y)\phi(y)dy, \tag{1}$$

for an eigenvalue λ and a corresponding eigenfunction $\phi(x)$ is considered in this paper. If λ is an eigenvalue of the kernel $K(x, y)$, then there is at least one non-null function $\phi(x)$ satisfying (1).

We shall discuss the case when $K(x, y)$ is a weakly singular kernel and has the form

$$K(x, y) = |x - y|^{-\alpha}, \quad 0 < \alpha < 1, \qquad \text{or } K(x, y) = \log|x - y|. \tag{2}$$

These two kernels are Hermitian and compact in $C[-1, 1]$, and hence have countable infinite numbers of eigenvalues with zero the only possible limit point (Atkinson [2]).

The main goal of the numerical methods to solve (1) is to reduce it approximately to an algebraic form. Then the solution of (1) is closely related to the solution of an algebraic eigenvalue problem.

The numerical treatment of an integral equation involving weakly singular kernel should take into account the nature of this singularity. Indeed, classical numerical methods such as product-integration, collocation and Galerkin methods,

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