

Original article

On asymptotic formulae via summability

P. Garrancho^a, D. Cárdenas-Morales^{a,*}, F. Aguilera^b

^a Departamento de Matemáticas, Universidad de Jaén, Campus Las Lagunillas s/n., 23071 Jaén, Spain

^b Departamento de Matemáticas, I.E.S. Virgen del Carmen, 23008 Jaén, Spain

Received 20 December 2009; received in revised form 23 June 2010; accepted 7 December 2010

Available online 16 December 2010

Abstract

In this paper we present a sort of Korovkin-type result on asymptotic formulae for sequences of linear operators not necessarily convergent but \mathcal{A} -summable, \mathcal{A} being a sequence of infinite matrices.

© 2010 IMACS. Published by Elsevier B.V. All rights reserved.

Keywords: Linear operators; Asymptotic formula; Summability; Almost convergence

1. Introduction

In 1948 Lorentz [13] introduced the notion of almost convergence:

- a bounded sequence of real numbers x_j is said to be almost convergent to ℓ if $L(x_j) = \ell$ for each Banach limit L , or equivalently if

$$\lim_{k \rightarrow \infty} \frac{1}{k} \sum_{j=n}^{n+k-1} x_j = \ell \quad \text{uniformly for } n \in \mathbb{N} = \{0, 1, 2, \dots\}. \quad (1)$$

In the seventies, within the general setting of the approximation of continuous functions f by means of sequences of linear positive operators $\{L_j\}$, King and Swetits [11] and Mohapatra [14] studied respectively qualitative and quantitative results on the almost convergence of the sequences $\{L_j f(x)\}$ ($L_j f(x)$ is the image of f by L_j evaluated at x) towards $f(x)$, and a few years later Swetits [16] unified these results and the classical ones (see [12,6]) by considering the following summability method introduced by Bell [3] in 1973:

* Corresponding author. Tel.: +34 953212144; fax: +34 953211871.

E-mail addresses: pgarran@ujaen.es (P. Garrancho), cardenas@ujaen.es (D. Cárdenas-Morales), pacoaguilera@iesvirgendelcarmen.com (F. Aguilera).