A study of plane wave propagation in anisotropic three-phase-lag and two-phase-lag model☆

Rajneesh Kumar *, Vijay Chawla

Department of Mathematics, Kurukshetra University Kurukshetra-136119, Haryana, India

1. Introduction

The Generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity. Hetnarski and Ignaczak [1] examined five generalizations of the coupled theory of thermoelasticity.

The first generalization is due to Lord and Shulman [2] who formulated the generalized thermoelasticity theory involving one thermal relaxation time. This theory is referred to as L-S theory or extended thermoelasticity theory in the Maxwell-Cattaneo law which replaces the Fourier Law of heat conduction by introducing a single parameter that acts as a relaxation time and obtained a wave-type equation by postulating a new law of heat conduction instead of classical Fourier's law. Green and Lindsay [3] developed a temperature rate-dependent thermoelasticity that includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction, when the body under consideration has a center of symmetry. One can refer to Hetnarski and Ignaczak [4] for a review and presentation of generalized theories of thermoelasticity.

Chadwick [5] and Chadwick [6] discussed propagation of plane harmonic waves in transversely isotropic and homogeneous anisotropic heat conduction solids respectively. Banerjee et al. [7] studied the thermoelastic waves in anisotropic solids. Four characteristic wave velocities are found, three being analogous to those of isothermal elastic waves. The fourth wave, which is predominately a temperature disturbance, corresponds to the heat pulses known as second sound.


The third generalization of the coupled theory of thermoelasticity is developed by Hetnarski and Ignaczak and is known as low-temperature thermoelasticity. This model is characterized by a system of non-linear field equations. Low-temperature non-linear models of heat conduction that predict wave like thermal signals and which are supposed to hold at low temperatures have also been proposed and studied in some works by Kosinski [15], Cimmelli and Kosinski [16].

The fourth generalization to the coupled theory of thermoelasticity introduced by Green and Naghdi and this theory is concerned with the thermoelasticity theory without energy dissipation, referred to as G-N theory of type II in which the classical Fourier law is replaced by a heat flux rate-temperature gradient relation. The heat transport equation does not involve a temperature rate term and as such this model admits undamped thermoelastic waves in thermoelastic material. The fourth generalization of the thermoelasticity theory involves a heat conduction law, which includes the conventional law and one that involves the thermal displacement gradient among the constitutive variables. This model is referred to as the G-N model 111 [17,18], which involves dissipation in general and admits thermoelastic waves.

The fifth generalization of the coupled theory of thermoelasticity is developed by Tzau [19] and Chandrasekhariah [20] and is referred to dual phase- lag thermoelasticity. Tzau [19] considered microstructural effects into the delayed response in time in the macroscopic formulation by taking into account that the increase of the lattice temperature is delayed due to phonon-electron interactions on the macroscopic level. A macroscopic lagging response between the temperature gradient and